Plasma Sources IV: High Density Sources: ICP, ECR, MW

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Outline

- Some Applications.
- Some general wave properties.
- ICP discharges.
- ECR discharges.
- Microwave discharges.
Some Applications
Application:
Etching of micro structures in silicon

Creation of ions and radicals and synergistic directional etching:
Reactive ion etching and plasma aided chemical vapor deposition.
Application: Deposition of Thin-Films
Some General Wave Properties
Basic Wave Equation

General wave equation (linear equation in $E!$):

$$\nabla^2 \ddot{E} - \dot{E}/c^2 = \mu_0 \dot{j} + \nabla \rho / \varepsilon_0 = \mu_0 \left( \sigma \dot{E} + \dot{j}_{ext} \right) + \nabla \rho / \varepsilon_0.$$  

The current in the plasma is proportional to the electric field:

$$\dot{j} = \sigma \cdot \vec{E}.$$  

The conductivity follows from the momentum equation.

No space charge for transversal waves:  
(only this case is considered here)

$$\rho / \varepsilon_0 = \nabla \cdot \vec{E} = ik \cdot \vec{E} = 0.$$  

• Only time varying currents contribute.  
• External currents $j_{ext}$ are source terms.  
• They can be included in the boundary conditions.  
• Without magnetic field, the conductivity $\sigma$ is isotropic,  
i.e. it is a scalar, with magnetic field it is a tensor.
Dispersion Relation

The wave equation together with the conductivity leads to the dispersion relation (for simplicity here scalar \( \sigma \)):

\[
(c k)^2 = i \omega \frac{\sigma}{\varepsilon_0} + \omega^2, \quad \text{using: } \varepsilon_0 \mu_0 = 1 / c^2
\]

The conductivity can be complex. Then also \( k \) is complex (for real \( \omega \)).

Example of a homogeneous (unmagnetized) plasma:

\[
\frac{\sigma}{\varepsilon_0} = \frac{e^2 n_e}{\varepsilon_0 m_e (\nu - i \omega)} = \frac{\omega_{pe}^2}{\nu - i \omega}
\]

- The imaginary part of the conductivity contributes to the real part of the wave vector.
- The real part of the conductivity contributes to the imaginary part of the wave vector.
Damping of the wave

Electric field: 

\[ E \propto \exp\left( i \left( \mathbf{k} \cdot \mathbf{r} - \omega t \right) \right) \]

Complex wave vector: 

\[ \mathbf{k} = \mathbf{k}' + i \mathbf{k}'' \]

Damped wave: 

\[ E \propto \exp\left( -\mathbf{k}'' \cdot \mathbf{r} \right) \exp\left( i \left( \mathbf{k}' \cdot \mathbf{r} - \omega t \right) \right) \]

- The imaginary part of \( k \) (real part of \( \sigma \)) leads to damping of the wave (\( k'' > 0 \)).
- In general, waves can also be excited in a plasma (\( k'' < 0 \)).
- This happens in case of instabilities.
Classical Skin Depth

Effective parameter:

$$w = \frac{\omega}{v_m}$$

Imaginary part:

$$k'' = \frac{\omega_{pe}}{c} \sqrt{\frac{w \left( w + \sqrt{1 + w^2} \right)}{2 \left( 1 + w^2 \right)}}$$

Real part:

$$k' = \frac{k''}{w + \sqrt{1 + w^2}} \leq k''$$

Skin depth (1/e amplitude depletion):

$$\delta_{skin} = 1 / k'' \Rightarrow E(z) \propto \exp\left( -z / \delta_{skin} \right)$$
Typical Parameters

Pressure $p = 10 \text{ Pa}$, hydrogen: \( n = 10^{11} \text{ cm}^{-3} \Rightarrow \omega_{pe} = 5.64 \cdot 10^4 \text{ s}^{-1} \sqrt{n} = 1.8 \cdot 10^{10} \text{ s}^{-1} \)

Plasma frequency: \( n = 10^{11} \text{ cm}^{-3} \Rightarrow \omega_{pe} = 5.64 \cdot 10^4 \text{ s}^{-1} \sqrt{n} = 1.8 \cdot 10^{10} \text{ s}^{-1} \)

(a) RF discharge: \( f = 13.56 \text{ MHz} \Rightarrow \omega = 0.85 \cdot 10^8 \text{ s}^{-1} \)

Effective parameter: \( w = 0.21 \ll 1 \)

Skin depth: \( \delta_{\text{skin}} \approx \frac{c}{\omega_{pe}} \sqrt{\frac{2}{w}} = 5.1 \text{ cm} \)

(b) MW discharge: \( f = 2.45 \text{ GHz} \Rightarrow \omega = 1.54 \cdot 10^{10} \text{ s}^{-1} \)

Effective parameter: \( w = 385 \gg 1 \)

Skin depth: \( \delta_{\text{skin}} \approx \frac{c}{\omega_{pe}} = 1.6 \text{ cm} \)
Effect of Collisions

Special cases:

\[
\delta_{\text{skin}} = \begin{cases} 
\frac{c}{\omega_{pe}} \left( \sqrt{\frac{2}{w}} \right) & \text{for } w > 1 \\
\frac{c}{\omega_{pe}} \left( \sqrt{\frac{2}{w}} \right) & \text{for } w < 1 \\
\frac{c}{\omega_{pe}} \left( \sqrt{\frac{2}{w}} \right) & \text{for } w \to \infty \quad \text{(HF case)} \\
\frac{c}{\omega_{pe}} \left( \sqrt{\frac{2}{w}} \right) & \text{for } w \to 0 \quad \text{(DC case)}
\end{cases}
\]

\[w = \frac{\omega}{\nu_m}\]

Collisions (dissipation) enlarge the skin depth.
Electric Field Profile

Note: Also the real part of $k$ depends on $w$.

- Without dissipation the wave is (classically) simply evanescent.
- With dissipation, damped oscillations can occur. (generally no practical consequences due to low amplitudes).
• With collisions, the evanescent wave has a low phase velocity that can be much smaller than \( c \).
• Without collisions, the phase velocity is infinite.
Failure of the Local Picture

- The classical approach presented before breaks down in a collisionless plasma (mean free path \( \lambda \gg \) skin depth \( \delta_{\text{skin}} \)).
- Then the conductivity is no longer a local quantity.
- For high collisionality, the local velocity depends only on the local field at the local time.
- Without collisions, it depends on the entire history, i.e. the trajectory in space and time.
- The correct description can no longer be performed in a fluid dynamic picture but requires a kinetic treatment, since the individual particle velocities matter.
- The entire interaction between wave and particles needs to be treated in parallel.
- This affects the skin depth and the power deposition!
Principle of the Non-Local Effect

- Thermal flux of electrons towards the antenna window.
- Reflection of the electrons at the floating potential sheath in front of the window.
- Gain and loss of energy occurs in different regions of space.
- Due to the field inhomogeneity in the skin layer, the net effect is non-zero.

Important:
The electric field oscillates perpendicular to the direction of the (thermal) particle motion and the field gradient!
Anomalous Skin Effect

- Alternation of positive and negative power deposition.
- Net effect is positive: Heating.

\[ \delta = \frac{8}{9\pi} \left( \frac{c^2 \mu}{\omega \sigma_0} \right) \] (mks)

Fig. 2. The field amplitude \( E \) as a function of normalized depth \( z = |\omega + \nu|/\nu \) for \( \alpha = 1 \) and \( \lambda = 0.3, 1, 10, 100 \).


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Consequences of the Non-Local Effect

• Net power deposition even without collisions: Stochastic heating.
• The field decay is not exponential.
• Different scaling of the skin depth (larger than classical):

\[
\delta_{\text{skin}} = \frac{8}{9} \frac{c}{\omega_{pe}} \left( \frac{\omega_{pe} v_{th}}{2 \omega c} \right)^{1/3} = \frac{8}{9} \left( \frac{c^2 v_{th}}{2 \omega \omega_{pe}^2} \right)^{1/3}
\]

\[
v_{th} = \sqrt{\frac{8kT_e}{\pi m}} \approx \text{typ.} 10^6 \text{ m/s}
\]

• Remarks:
  - The factor 8/9 is often skipped.
  - Requirement for stochastic heating:
  - Difficult to realize at low pressures.
  - Most effective at lower RF frequencies, e.g. 1 MHz.
Frequency and Pressure Dependence

- Experimental result from an ICP discharge.
- The ratio between the total (collisional) and the Ohmic overall power deposition (volume integrated) is determined.
- The scaling is similar for all frequencies.
- The non-local effect becomes effective only for pressures of less than 1 Pa.

Some General Technical Consequences

- The electron density in metals is much higher.
- Therefore the skin depth is much smaller.
- In copper typical values for RF frequencies are $10 \, \mu\text{m}$.
- For microwaves the skin depth is only about $1 \, \mu\text{m}$.
- The current is flowing only at the surface.
- Large surfaces are required.
- The cross section does not matter.
- The surface quality matters.
- Contacts can cause strong losses.
- Polishing and coating with silver are common measures.
- Further: Impedances scale with frequency.
- Dielectric isolators can easily transmit displacement current, tiny bends in wirings can cause noticeable inductances.
- Without impedance matching strong reflections occur.
ICP discharges
Basic Principles of CCP and ICP Discharges

CCP-mode: „piston“-principle

ICP-mode: transformer-principle
Radio Frequency (RF)

CCP and ICP discharges operate in the radio frequency regime:

• processing of non-conducting dielectrics (CCP)
• extra heating mechanism for electrons (CCP) (not existing in DC discharges)
• operation at low pressures (< 1 Pa)
• high plasma densities (up to $10^{12}$ cm$^{-3}$)
• large scale homogeneous discharge possible
• high efficiency for the transformer (ICP)

The most common frequency is 13.56 MHz (and harmonics).

This is not a magic but a legal number by international telecommunication regulations (also harmonics).
Some General Characteristics I

• ICP discharges operate usually at an order of magnitude higher powers than CCPs.
• They produce an order of magnitude more dense plasmas.
• The plasma potential is low and quiet for pure ICP operation.
• They can sometimes operate in a hybrid CCP/ICP mode, since the antenna can also act as an electrode (especially at low densities).
Some General Characteristics II

• Often they are operated together with a separate CCP: ICP for plasma generation, CCP for RF-bias.
• This allows independent control of the plasma density (ion flux) and the bias (ion energy).
• The antenna coil is usually either a cylindrical or planar coil.
• A dielectric window (quartz or aluminium oxide) is required.
Typical Antenna Coils

Cylindrical Antenna

Planar Antenna

**Faraday shield** (spoke like structure):
- only poloidal fields can penetrate
- capacitive coupling is suppressed.
- disadvantage: No self-ignition, high voltage between coil and the shield
ICP Matching Unit

![Diagram of ICP Matching Unit]

Graph showing capacitance (nF) vs. resistance (Ω) with curves for different values of resistance.

C1 and C2 are indicated on the graph.

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Calculated antenna fields in vacuum: $B_z$ and $E_\phi$

The relevant induced electric field is in azimuthal direction.
Induced Azimuthal Electric Field (calculated for vacuum)

The radial current is causing distortion of the symmetry.
Electric Field Induced by a Flat Coil Antenna

Flat coil with four parallel spirals, each 2.5 turns. (HV in the centre, ground at the outer connectors)

Figure 4. Oscillatory velocity amplitude (a) and relative phase (b) obtained from emission spectroscopy (RF-MOS) at a pressure of $p = 0.5$ Pa.

Wave Equation for the Induced Field

\[ \Delta \vec{E} = \frac{\partial^2 \vec{E}}{\partial z^2} + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \vec{E} \right) \right) = \frac{\omega^2}{c^2} \vec{E} - i \omega \mu_0 \vec{j}. \]

- Two dimensional problem.
- Displacement current can be neglected since the plasma frequency is much higher than the RF frequency.
- Analytical solution only for a homogeneous plasma.
- The flat antenna is approximated as a conducting disc of radius R.
- Plasma is assumed as homogeneous!
- Inhomogeneous plasma: Numerical solution!
Plasma Current Density

Local isotropic conductivity:

\[ \vec{j} = \sigma \vec{E} \]

\[ \sigma = \frac{e^2 n}{m (\nu + i\omega)}. \]

- For mean free paths of the order of the skin depth or larger the conductivity is no longer local.
- Then the product of conductivity and field becomes a convolution.
Geometry and Boundary Condition

Conditions are set at the walls and the interfaces:

\begin{align*}
  z &= 0 \\
  E_v \big|_{z=0} &= E_q \big|_{z=0}, \\
  \left. \frac{\partial E_q}{\partial z} \right|_{z=0} &= \left. \frac{\partial E_v}{\partial z} \right|_{z=0} = i \omega \mu_0 j_{\text{Antenne}}, \\
  z &= d \\
  E_q \big|_{z=d} &= E \big|_{z=d}, \\
  \left. \frac{\partial E_q}{\partial z} \right|_{z=d} &= \left. \frac{\partial E}{\partial z} \right|_{z=d}, \\
  \text{generally} \\
  E_v (z \to -\infty) &= 0, \\
  E (z = Z) &= 0 \quad \text{und} \\
  E (z = L) &= 0.
\end{align*}
Complex Solution for the Electric Field

Radially Bessel functions, axially exponentials:

\[ E(r, z) = \sum_{n=1}^{\infty} \tilde{A}_n \sinh \left( \sqrt{\left( \frac{\lambda_n}{L} \right)^2 + k_p^2 (Z - z)} \right) J_1 \left( \frac{\lambda_n}{L} \right) \]

The coefficients are:

\[ \tilde{A}_n = \frac{i \omega \mu_0 I N \pi}{\lambda_n} \frac{J_0 \left( \frac{\lambda_n}{L} \right) H_1 \left( \frac{\lambda_n}{L} \right) - J_1 \left( \frac{\lambda_n}{L} \right) H_0 \left( \frac{\lambda_n}{L} \right)}{J'_1 \left( \lambda_n \right)} e^{-\lambda_n \frac{d}{L}} \frac{\lambda_n \sinh \left( \sqrt{\left( \frac{\lambda_n}{L} \right)^2 + k_p^2 (Z - d)} \right) + \sqrt{\lambda_n^2 + (k_p L)^2} \cosh \left( \sqrt{\left( \frac{\lambda_n}{L} \right)^2 + k_p^2 (Z - d)} \right)}{\lambda_n \sinh \left( \sqrt{\left( \frac{\lambda_n}{L} \right)^2 + k_p^2 (Z - d)} \right) + \sqrt{\lambda_n^2 + (k_p L)^2} \cosh \left( \sqrt{\left( \frac{\lambda_n}{L} \right)^2 + k_p^2 (Z - d)} \right)} \]

\( \lambda_n \): n-th zero of the Bessel function

\[ k_p^2 = i \omega \mu_0 \sigma \]
Relevance of the Individual Terms

Only very few terms contribute, actually mainly 2!

Electric Field Depletion

- Approximately exponential decay of the field.
- The depletion length has a geometrical and a plasma contribution:

\[ \nu \ll \omega : \quad 1/\delta \approx \sqrt{\left(\frac{\lambda_1}{L}\right)^2 + \left(\frac{\omega_{pe}}{c}\right)^2}, \quad \lambda_1 = 3.83 \]

Typical values:

\[ \ell = \lambda_1 \frac{c}{\omega_{pe}} = \text{typ. } O\left(0.1m\right) \]

- Large chambers \((L \gg \ell)\): classical skin depth behaviour
- Small chambers \((L \ll \ell)\): geometrical depletion

- In reality, mostly a mixture of geometry and skin depth!
Radial Profile

- Fundamental radial mode profile is a Bessel function.
- Only the first two Bessel functions contribute significantly.
- Higher modes shift the maximum to smaller radii.
- Rule of thumb: Maximum at $r = 2/3 \, R$.

$$E(r) \approx A_1 J_1 \left( \lambda_1 \frac{r}{L} \right) + A_2 J_1 \left( \lambda_2 \frac{r}{L} \right)$$

Physics:
- B-field is a dipole.
- Naturally, no electric field on axis and no field at the wall.
- Zero at the conducting wall due to eddy currents.
Comparison with numerical integration

analytical solution

numerical solution (Comsol)

Self consistent numerical solution I

Plasma density and ion flux


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Self consistent numerical solution II

Induced power and electron temperature


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Collisional losses are strong in hydrogen and the energy is balanced locally. The electron temperature reproduces schematically the torus structure of the induced electric field (radial direction).
Phase Resolved Velocity Distribution Function in an ICP
(Ar, 0.5 Pa, 1 kW, Thomson Scattering)

Argon ICP discharge, $f = 13.56$ MHz, $P = 1$ kW, $p = 0.5$ Pa. Accumulation over 90,000 shots (30 minutes) at each phase. Temporally independent EEPF, oscillating drift velocity.

$\begin{align*}
  kT_e &= 2.6 \text{ eV} \\
  n_e &= 1.0 \times 10^{11} \text{ cm}^{-3}
\end{align*}$

Harmonically Oscillating Drift Velocity

\[ v = \hat{v} \sin(\omega t + \varphi) + v_0 \]

\[ \hat{v} = \frac{e \hat{E}}{m \omega} \quad \Rightarrow \quad \hat{E} = 0.66 \text{ V/cm} \]

Self consistent numerical solution III

Electric field amplitude and phase

Measurement (B-dot probe)

Very similar to simulation.

Optically Measured Field Penetration

Intensity and field are radially integrated and the axial dependence and time evolution are displayed as coloured contour plots.

Measurement (optical modulation)

Very good agreement throughout.

The plasma inductance and resistance are transformed to an effective antenna inductance and resistance.

\[
R_S = R_2 \cdot \frac{(\omega L_{12})^2}{R_2^2 + (\omega L_2)^2}
\]

\[
\omega L_S = \omega L_1 - \omega L_2 \cdot \frac{(\omega L_{12})^2}{R_2^2 + (\omega L_2)^2}
\]

\[
P_{\text{diss}} = \frac{1}{2} R_S \dot{I}_{RF}^2
\]

Low and High Density Operation

Transformed resistance:

Low density operation (skin depth >> plasma size):

\[ R_s = R_p \frac{(\omega L_p)^2}{R_p^2 + (\omega L_p)^2} \]

\[ R_p \gg \omega L_p \Rightarrow R_s \propto \frac{1}{R_p} \propto n_e \]

High density (normal) operation (skin depth << plasma size):

\[ R_p \ll \omega L_p \Rightarrow R_s \propto R_p \propto \frac{1}{\sigma_p \delta} \propto \frac{1}{\sqrt{n_e}} \]
Dissipated Power and Operation Threshold

Low density:

\[ P_{\text{diss}} \propto n_e \hat{I}_{\text{RF}}^2 \]

High density:

\[ P_{\text{diss}} \propto \frac{1}{\sqrt{n_e}} \hat{I}_{\text{RF}}^2 \]

Power lost by the discharge (collisions, flux to the wall):

\[ P_{\text{loss}} \propto n_e \]

- There is a minimum power (current) for ICP operation.
- Below this value, the discharge operates capacitively.
Capacitive Coupling by the Antenna

• The quartz window and the sheath form a non-linear capacitive voltage divider between the antenna and the plasma.

• At high densities and/or low antenna voltages, the capacitive coupling is reduced strongly.
Effect of CCP-ICP Mode Transition on the Matching

- Divergence of the matching capacitance $C_1$ at the transition point to CCP operation.

Effect on the Modulation of the Optical Emission

Discharge in hydrogen at $p = 10$ Pa:
mode transition CCP to ICP at $P = 150$ W

Observation of the Balmer–α emission line (integrated spatially):

$$\text{modulation} = \frac{I(t)}{\bar{I}} \cdot 100\%$$
Observation of the Mode Transition in the Modulation of the Optical Emission

ICP discharge in hydrogen at $p = 10$ Pa, Balmer–$\alpha$ emission
Pulsed mode: $P = 300$ W, $f = 1$ kHz, 1:1

• ICP discharges always ignite as CCPs.
• Then the density rises, the sheath shrinks, and the mode transition happens.

The mode transition between ICP and CCP has a strong effect on the ion energy distribution function.

- **ICP**: Quasi mono-energetic, low-energy ions from the floating potential sheath.
- **CCP**: High energy wide distributed ions from the RF sheath.
Further Topics in ICPs

1) Pulsed discharges and afterglow.
2) Electronegative discharges and instabilities.
3) Gas heating and electron pressure effects.
4) Atmospheric pressures.
5) Hybrid discharges ICP – CCP.
6) Many applications…..
ECR Discharges
(and a little bit of Whistlers, Helicons)
Addition of a Weak Axial Magnetic Field

Argon $p = 0.1$ Pa, $B = 10$ mT, $\lambda_{me} = 1$ m = 2 x diameter.

ICP, without B-field

Helicon, with B-field

Y. Celik et al., POP 18, 022107 (2011)
RF Modulation Spectroscopy Upstream

oscillatory velocity ($2\omega$)

Y. Celik et al., POP 18, 022107 (2011)

azimuthal drift velocity ($1\omega$)

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RF Modulation Spectroscopy Downstream

- Modulation can still be observed downstream.
- A helicon wave propagates along the field lines.
- The diamagnetic drift ($\text{grad}(T_e)$) displays the RF phase front.
- The phase front is bended due to the inhomogeneous dispersion by the density profile.

Y. Celik et al., POP 18, 022107 (2011)
- The antenna field determines the $B_z(r)$ wave field.
- Maxwell’s equations then determine the Helicon $B_\varphi(r)$:

\[
B_\varphi(r) \propto \frac{\partial}{\partial r} B_z(r)
\]

Y. Celik et al., POP 18, 022107 (2011)
Electromagnetic Fields Propagating Along Static Magnetic Fields

- The wave vector \( k \) points along the field lines.
- All electric fields are perpendicular to the wave vector \( k \).
- Two fundamental polarization states:
  - R-wave (rotating clockwise, with the electrons, “+”)
  - L-wave (rotating counter clockwise, with the ions, “-”)

\[
\vec{E} = \vec{E}_0 e^{i(kz-\omega t)} \begin{pmatrix} 1 \\ \pm i \end{pmatrix}
\]

- \( x \)- and \( y \)-components are 90° out of phase.
- The Lorentz force is important in this case:

\[
m\dot{v} = -\frac{e}{m} (\vec{E} + \vec{v} \times \vec{B})
\]

- Electrons rotate clockwise with the cyclotron frequency:

\[
\omega_c = \frac{eB}{m_e}
\]
Current density from the momentum equation:

\[ -i \omega \vec{j} = \frac{\omega_{pe}^2}{c^2} \frac{\vec{E}}{\mu_0} - \omega_c \vec{j} \times \vec{e}_z \]

Solution of the equation (determines also the conductivity):

\[ -i \omega \mu_0 \vec{j} = \frac{\omega_{pe}^2}{c^2} \frac{1}{1 + \frac{\omega_c}{\omega}} \vec{E} \]

Insertion into the wave equation gives the dispersion relation:

\[ -k^2 \vec{E} + \frac{\omega^2}{c^2} \vec{E} = -i \omega \mu_0 \vec{j} \quad \Rightarrow \quad -k^2 + \frac{\omega^2}{c^2} = \frac{\omega_{pe}^2}{c^2} \frac{1}{1 + \frac{\omega_c}{\omega}} \]
Structure of the Dispersion Relation

Solving for $k^2$ yields:

$$(ck)^2 = \omega^2 - \frac{\omega_{pe}^2}{1 \mp \frac{\omega_c}{\omega}} \quad "-": R - \text{wave, } "+": L - \text{wave}$$

Normalization by the electron plasma frequency helps identifying the characteristics:

$$k^2 = \omega^2 - \frac{1}{1 \mp \frac{\omega_c}{\omega}}$$

mit $c k / \omega_{pe} \rightarrow k$, $\omega / \omega_{pe} \rightarrow \omega$, $\omega_c / \omega_{pe} \rightarrow \omega_c$
Cut-Offs and Resonances

Cut-off frequencies \((k = 0)\) are easily identified:

\[
0 = \omega^2 - \frac{1}{1 \mp \frac{\omega_c}{\omega}} \quad \Rightarrow \quad \omega = \omega_{R/L} = \frac{1}{2} \left( \pm \omega_c + \sqrt{\omega_c^2 + 4} \right), 0
\]

\[
\Rightarrow \Delta \omega = \omega_R - \omega_L = \omega_c
\]

- Three intersections with the \(\omega\)-axis.
- R-wave at \(\omega = 0\) and \(\omega = \omega_R\).
- L-wave at \(\omega_L\), in between the two cut-offs of the R-wave.
- Only the R-wave has a resonance \((k \rightarrow \infty)\).
- This resonance is at the cyclotron frequency \(\omega = \omega_c\).
- Natural result with view on the electron gyration.
R- and L-Wave Dispersion

- The dispersion relation is shown for $\omega_c = 2 \omega_{pe}$.
- Then the L-wave cut-off is below the R-wave resonance ($\omega_c$).
- In case $\omega_c < \omega_{pe}$, the L-wave cut-off is between $\omega_R$ and $\omega_c$.
- The R-wave has a band-gap between $\omega_c$ and $\omega_R$.
- The L-wave has band-gap between 0 and $\omega_L$.
- For large $k$ both waves behave like waves in vacuum with a phase velocity $c$. 

![Graph showing R and L wave dispersion relations](image)
R-Wave Dispersion at Low $\omega$ and $k$: Whistler Waves

- At low frequencies only R-waves exist (cut-off for L-waves).
- The dispersion relation can then be expanded to first order ($\omega \ll \omega_c$) which provides a simple expression for the phase velocity:

$$V_{ph} = \frac{\omega}{k} \approx c \frac{\sqrt{\omega \omega_c}}{\omega_{pe}}$$

- The phase velocity increases monotonically with $\omega$.
- Higher frequencies propagate faster and arrive earlier.
- If all frequency start at the same time at $x = 0$ and detection is after a distance $s$, each frequency arrives at its own time:

$$t = \int_0^s \frac{dx}{V_{ph}} \propto \frac{1}{\sqrt{\omega}} \Rightarrow \omega(t) \propto \frac{1}{t^2}$$
Natural Whistlers

- Lightning flashes at the southern hemisphere excite whistler waves within a broad spectral range (kHz).
- Waves propagate along the earth magnetic field lines.
- Due to dispersion, higher frequencies arrive earlier.

A falling glissando can be heard (coining the term “whistler”).
- The time dependent frequency spectra scale like $1/t^2$.
- A fit “by eye” shows very good agreement (red lines).

Stanford vfl group

http://www-pw.physics.uiowa.edu/mcgreevy/

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Helicon and Whistler Dispersion

• Helicon waves are whistler waves which are radially bounded.
• This requires a finite $k_r$ and the dispersion relation changes.

Comparison:

$$\frac{\omega}{\omega_c} \left( \frac{\omega_{pe}}{c k_r} \right)^2 = \left\{ \begin{array}{l}
\frac{k_z}{k_r} \sqrt{1 + \left( \frac{k_z}{k_r} \right)^2} \\
\left( \frac{k_z}{k_r} \right)^2
\end{array} \right.$$

Helicon

Whistler

Note: $k^2 = k_z^2 + k_r^2$

• The difference is particularly pronounced at $R < \lambda_z$.
• For large radii the Helicon wave eventually converges to the normal whistler wave.
Faraday Rotation

• At high frequencies R- and L-waves both travel almost with the speed of light in vacuum.
• However, still the R-wave is a little faster.
• Therefore, with distance $L$ an increase phase difference $\Delta \varphi$ develops. In second order in $\omega_{pe}/\omega$ this difference is:

$$\Delta \varphi = \varphi_L - \varphi_R = L \Delta k = \frac{L \omega_c}{c} \left( \frac{\omega_{pe}}{\omega} \right)^2 \propto B_0 n$$

Transformation of a linearly polarized wave into a left and right polarized wave:

$$\left( \hat{e}_x + i \hat{e}_y \right) + \left( \hat{e}_x - i \hat{e}_y \right) = 2 \hat{e}_x$$

• The phase difference between the R- and L-components rotates the direction of polarization.
• This effect is called Faraday rotation.
Linear Polarization Rotation

The R- and L- parts are recombined after the wave has experienced a certain phase shift:

\[
\vec{E}_R + \vec{E}_L = \frac{E_0}{2} e^{i\varphi_R} \left[ \begin{pmatrix} 1 \\ i \end{pmatrix} + e^{i\Delta\varphi} \begin{pmatrix} 1 \\ -i \end{pmatrix} \right] = E_0 e^{i\varphi_R + \Delta\varphi/2} \begin{pmatrix} \cos(\Delta\varphi/2) \\ -\sin(\Delta\varphi/2) \end{pmatrix}
\]

The initially only x-direction polarized wave has been rotated by an angle \(-\Delta\varphi/2\) in direction of the y-axis.
Application of Faraday Rotation

Applications:

- Measuring magnetic fields in astro physics.
- Determination of magnetic fields in fusion experiments.
- Building optical diodes for laser applications.

B.M. Gaensler et al., Science 307m 1610 (2005)
Large Magellain Cloud

Figure 1. Schematic drawing of the HCN-interferometer/polarimeter. The new channel measures the Faraday rotation angle and the line-integrated electron density along a horizontal line of sight in the equatorial plane. The beat signals are detected with He-cooled InSb detectors giving a good signal-to-noise ratio and a high time resolution. The beat frequency for this channel can be increased by a second rotating grating, thus allowing a time resolution up to 10 \( \mu \)s.

H.R. Koslowski, H. Soltwisch, W. Stodiek,
Plasma Physics and Controlled Fusion 38, 271 (1996)
Electron Cyclotron Resonance (ECR)

- ECR discharges take advantage of the resonance of the R-wave at the electron cyclotron frequency.
- For commercial applications they are generally operated at a microwave frequency of \( f = 2.45 \text{ GHz} \).
- This requires a magnetic field strength of \( B = 87.5 \text{ mT} \).
- The magnetic field is generally non-uniform.
- Waves are launched from the high field side \( (\omega_c > \omega) \).

Examples of ECR Sources

- Ion source for highly charged ions, e.g. at accelerators.
- High density plasma source in the semiconductor industry.
- Satellite propulsion.

THE SACLAY (FRANCE) HIGH-CURRENT PROTON AND DEUTERON ECR SOURCE

Roth & Rau: commercial ECR source
z.B. for semiconductor treatment,
At $P = 1$ kW, $n = \text{a few } 10^{17} \text{ m}^{-3}$.
Pressure: $p = 0.1 – 10$ Pa.

U. Czarnetzki, ISLTPP, Bad Honnef 2016
Control of the Resonance Location by the Magnetic Field

Schematic of ECR Chamber and Two Solenoid Magnet

Plasma Density vs Magnetic Field

POSTECH (Korea)
* Work supported by KAERI’s KOMAC Project
ECR Heating (ECRH) at the New STELLARATOR Wendelstein 7-X (W7-X)

1 MW at $f = 140$ GHz

Figure shows a so called “gyratron”, a high power microwave source. So far 10 gyratrons installed at Wendelstein with a total power of 10 MW.

Fig. 2 Prototype Gyrotron ("Maquette") during test (by courtesy of FZK)
Ion Cyclotron Resonance Heating (ICRH)

ICRH (Ion Cyclotron Resonant Heating) at the TOKAMAK Joint European Torus (JET): 23 – 57 MHz, up to 32 MW in 16 channels of 2 MW.
Collisionless Heating in ECR Discharges

The equation of motion for a single particle is solved:

\[ m\ddot{\vec{v}} = -e\left(\vec{E}_0 + \vec{v} \times \vec{B}\right), \quad \vec{v}(z = z_0, t = 0) = \vec{v}_0 \]

- The time varying electric field \( E \) is assumed as homogeneous, i.e. it is only a function of time (corrections -> later).
- The static magnetic field \( B \) points only in the \( z \)-direction and is only a function of \( z \) (magnetic mirror effects are ignored).
- The aim is the calculation of the change in energy \( \Delta \varepsilon \) between the initial moment and a later moment.
Free Gyrating Particle

- The initial condition can always be met by adding the solution for a free gyrating electron.

\[ m\ddot{v}_f = -e\ddot{v}_h \times \vec{B}, \quad \dot{v}_f (z = z_0, t = 0) = \dot{v}_0 \]

- This solution cancels out later when calculating the energy change and averaging over all possible initial phases.

- Solution for a homogeneous \( B \) field:

\[ \dot{v}_f = v_0 \left( \sin(\vartheta_0)\cos(\omega_c t + \varphi_0) \right) \]

\[ \left( \begin{array}{c}
\sin(\vartheta_0) \\
-\sin(\vartheta_0) \\
\cos(\vartheta_0)
\end{array} \right) \]

\[ \omega_c = \frac{eB}{m} \]

- If \( B \) varies with \( z \) only the time dependence changes.

- **Note:** Ions gyrate counter clockwise and have a much higher mass (lower frequency).
Homogeneous $B$-Field

- The fields and initial conditions are now:
  \[
  \frac{e \mathbf{B}}{m} = \omega \hat{e}_z, \quad \mathbf{E} = E_0 \left( \hat{e}_x + i \hat{e}_y \right) e^{-i \omega t}, \quad \mathbf{v}(t=0) = 0
  \]

- The solution is:
  \[
  \mathbf{v} = \mathbf{v} \varphi \begin{pmatrix} \cos(\varphi) \\ -\sin(\varphi) \\ 0 \end{pmatrix}, \quad \varphi = \omega t, \quad \mathbf{v} = e \frac{E_0}{m \omega}
  \]

- The velocity amplitude increases linearly with time!
- The energy increase quadratically:
  \[
  \langle \Delta \varepsilon \rangle = \frac{m \mathbf{v}^2}{2} \varphi^2
  \]

- Therefore, very high energies can be reached.
Inhomogeneous $B$ Field

- An inhomogeneous $B$ field is now approximated by a linear variation (first order expansion around the resonance point):

$$\frac{e \tilde{B}}{m} = \omega \left( 1 - \frac{z}{L} \right) \tilde{e}_z$$

- The solution is slightly more tricky but in the end effectively the linear term $\varphi$ is replaced by an integral and some phase shift appears:

$$\tilde{v} = \hat{v} \Psi(\varphi) \left( \cos\left( \varphi - \pi/4 + \theta(\varphi) \right) \right)$$

$$\tilde{v} = \hat{v} \Psi(\varphi) \left( \cos\left( \varphi + \pi/4 + \theta(\varphi) \right) \right), \quad \varphi = \omega t, \quad \hat{v} = \frac{e E_0}{m \omega}$$

- The integral reads:

$$\Psi(\varphi) = \left| \int_{0}^{\varphi} e^{i \Omega(\varphi')} d\varphi' \right| \approx \sqrt{2 \pi \frac{L \omega}{v_0 z}}, \quad \Omega(\varphi) = \frac{v_0 z}{2 L \omega} \varphi^2 + \frac{z_0}{L} \varphi$$

- For typical parameters, the integral converges to a constant:

$$L \omega >> v_0 z \Rightarrow 10^{-2} m \cdot 10^{10} s^{-1} >> 10^6 m/s$$
Energy Gain Along the z-Axis

Since the velocity along z is constant, phase $\varphi$ and $z$-position can be directly converted:

$$\frac{z}{L} = \frac{v_{0z}}{L \omega} \varphi - \frac{z_0}{L}$$

Naturally, the width of the resonance zone is about $L$. 
Energy Gain in an Inhomogeneous $B$-Field

• The result for the energy gain is then:

$$\langle \Delta \epsilon \rangle = \frac{m \hat{v}^2}{2} \left( 2\pi \frac{L \omega}{v_{0z}} \right) \text{ with } \frac{L \omega}{v_{0z}} \gg 1, \quad \hat{v} = \frac{eE_0}{m \omega}$$

• The gain scales inversely with the initial velocity in $z$-direction, i.e. it is proportional to the transit time through the resonance zone.

• The power per unit area is obtained by multiplying with the flux density $n v_{0z}$:

$$S = \frac{\pi \left( eE_0 \right)^2 Ln}{m \omega}$$

• Remarkably, this result is independent of the initial velocity.

• Since the frequency is very high collisions do not play a significant role up to a quite high pressure of about 100 Pa.
Wave Equation Model

- A correct description of the heating process must take the propagation and depletion of the wave into account.
- The collisionless wave equation is (R - dispersion relation):

\[ \frac{\partial^2 E_r}{\partial z^2} + k_0^2 \left( 1 - \frac{\omega_{pe}^2}{\omega(\omega - \omega_c(z))} \right) E_r = 0, \quad k_0 = \frac{\omega}{c} \]

- Using again the linear B-field and constant plasma density approximation and some normalization, the equation reads:

\[ \frac{\partial^2 E_r}{\partial \xi^2} + \left( 1 - \frac{\eta}{\xi} \right) E_r = 0, \quad \xi = zk_0, \quad \eta = \frac{\omega_{pe}^2 L}{\omega c} \]

- A solution to this equation was found by K.G. Budden in 1966 but it is rather complex.

K.G. Budden, Radio Waves in the Ionosphere, Cambrige University Press, UK 1966
Reflection and Transmission

• One of the most interesting aspect about the solution is that it allows also calculation of the reflection and transmission coefficients. These results are quite simple:

\[
\frac{S_{abs}}{S_{inc}} = 1 - e^{-\pi \eta}, \quad \frac{S_{trans}}{S_{inc}} = e^{-\pi \eta}, \quad \frac{S_{ref}}{S_{inc}} = 0, \quad \eta = \frac{\omega_{pe}^2 L}{\omega c} = \frac{\omega_{pe}^2}{\omega^2} k_0 L
\]

• Quite unexpected, there is no reflection.

• Typical values are \(k_0 = 50 \text{ m}^{-1}\), \(L = 0.1 \text{ m}\) which leads to a moderate density condition:

\[
\frac{\omega_{pe}^2}{\omega^2} \geq 0.2 \implies n \geq 1.5 \cdot 10^{10} \text{ cm}^{-3}
\]
Tunneling Through the Band Gap

- The result of Budden implies that the wave is tunneling through the band gap which exists between $\omega_c$ and $\omega_R$.
- Since $\omega_c$ and $z$ are interchangeable variables in the model, it is illustrative to re-plot the dispersion relation in an alternative representation.
- For low plasma densities the band gap is small and the wave can tunnel.
- Within the band gap, the wave is evanescent.
Further Aspects

• Doppler shift of the resonance:

\[ \omega + k(z)v_z = \omega_c(z) \]

• This shifts the resonant magnetic field by typically a few percent.
• Heating at higher harmonics of the cyclotron frequency.
• Parametric instabilities and non-linear power absorption.
• Confinement of electrons in a mirror field.
• Effect of the chamber geometry on resonances (cavity effect at longer wavelengths or smaller chambers).
• Effects related to the transversal magnetic field.
Microwave Discharges
Some Advantages of Microwave Sources

- Simplicity of plasma generation at high (>100 W/cm$^3$) and low (< 1 W/cm$^3$) powers.
- Wide range of operating pressures (from 10 Pa up to atmospheric pressure).
- Control of the density profile by the antenna structure.
- Size of the discharge chamber can be variable.
- No electrodes.
- Large areas / volumes can be treated.
- High power sources at relatively low price available.

Operation Schemes

For most microwave sources the operation principle is either:
- Standing waves in a resonator.
- Propagation of surface waves (with or without resonator).

A. Schulz et al., Contrib. Plasma Phys. 52, 607 2012

Plasma torch:
Surface Waves

- Dielectric \((x > 0)\) - plasma \((x < 0)\) interface.
- Wave is evanescent on both sides (exponential decay).
- Propagation along the surface \((z – direction)\).
- Wave guide effect.
- Simplification: Homogeneous plasma.

\[ E_x, E_z, B_y \propto \exp(-\kappa m |x|) \exp(i k_m z) \exp(i \omega t) \]
\[ m = P, D \]
\[ P(x < 0): \varepsilon_p = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i \nu)} = 1 + i \frac{\sigma}{\varepsilon_0 \omega} \]
\[ D(x > 0): \varepsilon_D \]

- Transversal electric and magnetic fields \((y – direction)\).
- Longitudinal \((z)\) electric field due to waveguide effect!
Surface Wave Parameters

Boundary condition (x = 0):

\[(i) \ E_z^{(P)} = E_z^{(D)} \iff \frac{1}{\epsilon_p} \frac{\partial B_y^{(P)}}{\partial x} = \frac{1}{\epsilon_D} \frac{\partial B_y^{(D)}}{\partial x}\]

\[(ii) \ B_y^{(P)} = B_y^{(D)}\]

- The boundary conditions couple the wave properties in the two regions.

Solution (Note: The quantities are complex due to collisions!):

\[k_p = k_D = k = \frac{\omega}{c} \sqrt{\frac{\epsilon_P \epsilon_D}{\epsilon_P + \epsilon_D}}\]

\[K_m = \frac{\omega}{c} \sqrt{\frac{-\epsilon_m^2}{\epsilon_P + \epsilon_D}}\]
Dispersion Relation (collisionless)

Simple case (no collisions):

\[ \epsilon_p = 1 - \frac{\omega^2}{\omega_{pe}^2} \]

\[ \frac{ck}{\sqrt{\epsilon_D \omega_{pe}}} = \sqrt{\frac{\frac{\omega^2}{\omega_{pe}^2} - 1}{1 + \epsilon_D - \frac{\omega_{pe}^2}{\omega^2}}} \]

\[ \approx \frac{\omega}{\omega_{pe}} \begin{cases} 
\omega \gg \omega_{pe} : \frac{1}{\sqrt{1 + \epsilon_D}} \\
\omega \ll \omega_{pe} : 1
\end{cases} \]

Resonance \( (\omega = \omega_r : k \to \infty) \):

\[ \frac{\omega_r}{\omega_{pe}} = \frac{1}{\sqrt{1 + \epsilon_D}} \]
Damping Evanescent Wave (collisonless)

- The decay of the wave in the plasma is always faster than in the dielectric. (This can change in cylindrical geometry.)
Sources Types I

- Cavity and waveguide sources: Typically for flow-tube reactors with a quartz or ceramic tube.

Figure 2. Cavity microwave plasma generators

Figure 3. Waveguide microwave plasma generators

Sources Types II

- Surface wave discharges: Flow tube as well as large area application.

Figure 4. Different designs of surface wave devices on the base of cylindrical (a, d) rectangular (b, c) and coaxial structures (e), duo-plasmaline ([10] Pr7-99) (f), large area surface wave plasma source (g) ([11] p. 163)

Resonator and Surface Wave: Surfatron

- Often used as atomic radical source, e.g. H, N, O etc. (and excited molecules).


https://plasimo.phys.tue.nl/applications/surfatron/index.html
Inverted Geometry: Duo Plasma Line

- Inner conductor, outer plasma.
- MW feeding from both ends to compensate for damping of the wave along the line.
- Large homogeneous (along the line) plasmas.
- Usually operated with a number of lines in parallel.

Sources Types III

- Large area and large volume source: Many slots with individual surface waves at each slot.
- Planar or ring shaped geometry.

Figure 5. (a) Waveguide holey-plate plasma generator ([11] p.175 ) and (b) SLAN-system (slotted antennas) ([10] Pr7-1)

The SLAN Source

Dirk Luggenhölscher, Dissertation RUB, 2004
Density Decay in the Afterglow

Argon dominated (90 % Ar, 10 % H₂): \( p = 30 \text{ Pa} \), \( f = 200 \text{ Hz} \), 20 % on

- Source terms only at the wall due to small skin depth (cm).
- Collisional electron cooling due to high pressure (cm).
- Diffusion of recombining plasma leads to hollow profiles.
- In the afterglow, relaxation to the basic diffusion mode.

Dirk Luggenhölscher, Dissertation RUB, 2004

\[ n(r) = n_0 J_0 \left( \frac{2.41 r}{R} \right) \]
Some Conclusions

• At high frequencies the penetration of the electromagnetic wave into the plasma is a major issue.
• ECR discharges allow the wave to travel long distances before the energy is deposited within a relatively short range.
• In ICP and microwave discharges the skin depth is short and power is deposited in the vicinity of the antenna.
• Particularly microwave discharge have a skin depth of typically only one cm. However, power can be distributed by operation of a large number of antenna slots. These are often connected to one common resonator.
• Microwaves can propagate in the sheath between surfaces and the plasma via surface waves.
• All discharges can produce dense plasmas in the range of typically \( n = 10^{10} \, \text{cm}^{-3} \) to \( 10^{13} \, \text{cm}^{-3} \) and electron temperatures of a couple of eV and much higher in ECRs.
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