Plasma Sources IV: High Density Sources: ICP, ECR, MW U. Czarnetzki

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Institute for Plasma and Atomic Physics

#### Outline

- Some Applications.
- Some general wave properties.
- ICP discharges.
- ECR discharges.
- Microwave discharges.

#### **Some Applications**

#### Application: Etching of micro structures in silicon



Creation of ions and radicals and synergistic directional etching: Reactive ion etching and plasma aided chemical vapor deposition.

#### **Application: Deposition of Thin-Films**





#### **Some General Wave Properties**

#### **Basic Wave Equation**

General wave equation:  $\nabla^2 \vec{E} - \ddot{\vec{E}}/c^2 = \mu_0 \dot{\vec{j}} = \mu_0 \left(\underline{\underline{\sigma}} \, \dot{\vec{E}} + \dot{\vec{j}}_{ext}\right)$ 

The current in the plasma is proportional to the electric field:  $\vec{j} = \underline{\sigma} \vec{E}$ 

The conductivity follows from the momentum equation.

No space charge for transversal waves: (only this case is considered here)

$$\nabla \cdot \vec{\mathbf{E}} = i\vec{k} \cdot \vec{\mathbf{E}} = 0.$$

- Only time varying currents contribute.
- External currents are source term.
- They can be included in the boundary conditions.
- Without magnetic field the conductivity  $\sigma$  is isotrop (scalar).
- With magnetic field it is a tensor.

#### **Dispersion Relation**

The wave equation together with the conductivity leads to the dispersion relation (for simplicity here scalar  $\sigma$ ):

$$(ck)^2 = i\omega \frac{\sigma}{\varepsilon_0} + \omega^2$$

The conductivity can be complex. Then also *k* is complex (for real  $\omega$ ). Example of a homogeneous  $\frac{\sigma}{\varepsilon_0} = \frac{e^2}{\varepsilon_0 m_e}$ 

$$\frac{\sigma}{\varepsilon_0} = \frac{e^2 n_e}{\varepsilon_0 m_e (v - i \omega)} = \frac{\omega_{pe}^2}{v - i \omega}$$

- The imaginary part of the conductivity contributes to the real part of the wave vector.
- The real part of the conductivity contributes to the imaginary part of the wave vector.

## Damping of the wave

Electric field: 
$$E \propto \exp\left(i\left(\vec{k} \cdot \vec{r} - \omega t\right)\right)$$

Complex wave vector:  $\vec{k} = \vec{k}' + i \vec{k}''$ 

Damped wave:  $E \propto \exp(-\vec{k} \cdot \vec{r}) \exp(i(\vec{k} \cdot \vec{r} - \omega t))$ 

- The imaginary part of k (real part of σ) leads to damping of the wave (k' > 0).
- In general, waves can also be excited in a plasma (k'' < 0).
- This happens in case of instabilities.

#### **Classical Skin Depth**



Skin depth (1/e amplitude depletion):

$$\delta_{skin} = 1/k'' \implies E(z) \propto \exp(-\delta_{skin}z)$$

#### **Typical Parameters**

Pressure p = 10 Pa, hydrogen:  $v_m = 4 \cdot 10^8 s^{-1}$ Plasma frequency:  $n = 10^{11} cm^{-3} \Rightarrow \omega_{ne} = 5.64 \cdot 10^4 s^{-1} \sqrt{n} = 1.8 \cdot 10^{10} s^{-1}$ (a) RF discharge:  $f = 13.56 MHz \Rightarrow \omega = 0.85 \cdot 10^8 s^{-1}$ Effective parameter: *w*=0.21<<1 Skin depth:  $\delta_{skin} \approx \frac{c}{\omega} \sqrt{\frac{2}{w}} = 5.1 cm$ (b) MW discharge:  $f = 2.45 GHz \Rightarrow \omega = 1.54 \cdot 10^{10} s^{-1}$ Effective parameter: w=385 >>1Skin depth:  $\delta_{skin} \approx \frac{c}{\omega_{rec}} = 1.6 \, cm$ 

#### **Effect of Collisions**

Special cases:

$$\delta_{skin} = \frac{c}{\omega_{pe}} \begin{cases} 1 & for \ w \gg 1 \\ \sqrt{\frac{2}{w}} & for \ w < 1 \end{cases} = \begin{cases} \frac{c}{\omega_{pe}} & for \ w \to \infty \ (HF \ case) \\ \infty & for \ w \to 0 \ (DC \ case) \end{cases}$$
$$w = \frac{\omega}{v_m}$$
Collisions (dissipation) enlarge the skin depth.

m

#### **Electric Field Profile**

#### Note: Also the real part of *k* depends on *w*.



- Without dissipation the wave is (classically) simply evanescent.
- With dissipation, damped oscillations can occur. (generally no practical consequences due to low amplitudes).

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- With collisions, the evanescent wave has a low phase velocity that can be much smaller than c.
- Without collisions, the phase velocity is infinite.

#### **Failure of the Local Picture**

- The classical approach presented before breaks down in a collisionless plasma (mean free path  $\lambda >>$  skin depth  $\delta_{skin}$ ).
- Then the conductivity is no longer a local quantity.
- For high collisionality, the local velocity depends only on the local field at the local time.
- Without collisions, it depends on the entire history, i.e. the trajectory in space and time.
- The correct description can no longer be performed in a fluid dynamic picture but requires a kinetic treatment, since the individual particle velocities matter.
- The entire interaction between wave and particles needs to be treated in parallel.
- This affects the skin depth and the power deposition!

#### **Principle of the Non-Local Effect**

- Thermal flux of electrons towards the antenna window.
- Reflection of the electrons at the floating potential sheath in front of the window.
- Gain and loss of energy occurs in different regions of space.
- Due to the field inhomogeneity in the skin layer, the net effect is non-zero.



#### **Anomalous Skin Effect**

THE PHYSICS OF FLUIDS

VOLUME 10, NUMBER 4

APRIL 1967

#### Anomalous Skin Effect in a Plasma

ERICH S. WEIBEL Laboratoire de Recherches en Physique des Plasmas, Lausanne, Switzerland (Received 10 August 1966)

The theory of the skin effect in a plasma is developed for the case in which the relation between current and electric field is not local.







- Alternation of positive and negative power deposition.
- Net effect is positive: Heating.



Godyak V A and Kolobov V I 1997 Negative power absorption in inductively coupled plasma *Phys. Rev. Lett.* 79 4589–92.

#### **Consequences of Non-Local Effect**

- Net power deposition even without collisions: Stochastic heating.
- The field decay is not exponential.
- Different scaling of the skin depth (larger than classical):

$$\delta_{skin} = \frac{8}{9} \frac{c}{\omega_{pe}} \left(\frac{\omega_{pe} v_{th}}{2 \omega c}\right)^{1/3} = \frac{8}{9} \left(\frac{c^2 v_{th}}{2 \omega \omega_{pe}^2}\right)^{1/3}$$
$$v_{th} = \sqrt{\frac{8 kT_e}{\pi m}}$$

- Remarks:
  - The factor 8/9 is often skipped.
  - Requirement for stochastic heating:
  - Difficult to realize at low pressures.
  - Most effective at lower RF frequencies, e.g. 1 MHz.

 $\frac{\omega c}{\omega_{pe} v_{th}} \ll 1, At f = 13.56 MHz:$ 

$$\Rightarrow \omega_{pe} >> 2 \cdot 10^{10} \, s^{-1}$$
$$\Rightarrow n_e >> 1 \cdot 10^{11} \, cm^{-3}$$

#### **Frequency and Pressure Dependence**

- Experimental result from an ICP discharge.
- The ratio between the total (collisional) and the Ohmic overall power deposition (volume integrated) is determined.
- The scaling is similar for all frequencies.
- The non-local effect becomes effective only for pressures of less than 1 Pa.



V. A. Godyak, R-B. Piejak, "Experimental Evidence of Collisionless Power Absorption in Inductively Coupled Plasmas", Physical Review Letters, 1998

#### **Some General Technical Consequences**

- The electron density in metals is much higher.
- Therefore the skin depth is much smaller.
- In copper typical values for RF frequencies are 10  $\mu$ m.
- For microwaves the skin depth is only about 1  $\mu$ m.
- The current is flowing only at the surface.
- Large surfaces are requires.
- The cross section does not count.
- The surface quality matters.
- Contacts can cause strong losses.
- Polishing and coating with silver are common measures.
- Further: Impedances scale with frequency.
- Dielectric isolators can easily transmit displacement current, tiny bends in wirings can cause noticeable inductances.
- Without impedance matching strong reflections occur.

#### **ICP discharges**

#### **Basic Principles of CCP and ICP Discharges**



# **Radio Frequency (RF)**

CCP and ICP discharges operate in the radio frequency regime:

- processing of non-conducting dielectrics (CCP)
- extra heating mechanism for electrons (CCP) (not existing in DC discharges)
- operation at low pressures (< 1 Pa)
- high plasma densities (up to  $10^{12}$  cm<sup>-3</sup>)
- large scale homogeneous discharge possible
- high efficiency for the transformer (ICP)

The most common frequency is 13.56 MHz (and harmonics).

This is not a magic but a legal number by international telecommunication regulations (also harmonics).

#### **Some General Characteristics I**

- ICP discharges operate usually at an order of magnitude higher powers than CCPs.
- They produce an order of magnitude more dense plasmas.
- The plasma potential is low and quiet for pure ICP operation.
- They can sometimes operate in a hybrid CCP/ICP mode, since the antenna can also act as an electrode (especially at low densities).

#### **Some General Characteristics II**

- Often they are operated together with a separate CCP: ICP for plasma generation, CCP for RF-bias.
- This allows independent control of the plasma density (ion flux) and the bias (ion energy).
- The antenna coil is usually either a cylindrical or planar coil.
- A dielectric window (quartz or aluminium oxide) is required.



# **Typical Antenna Coils**

#### Cylindrical Antenna



#### Planar Antenna



Faraday shield (spoke like structure):

- only poloidal fields can penetrate
- capacitive coupling is suppressed.
- disadvantage: No self-ignition, high voltage between coil and the shield



## **ICP Matching Unit**





#### **ICP Version of the GEC Reference Cell**





d = 5 cm, r = 5 cm

typical discharge conditions:

- f = 13.56 MHz
- p = 0.1 10 Pa
- P = 10 1000 W
- $n_e = 10^{10} 10^{12} \text{ cm}^{-3}$

$$T_e = 1 - 5 eV$$

# Calculated antenna fields in vacuum: $B_z$ and $E_{\phi}$



The relevant induced electric field is in azimuthal direction.

#### Induced Azimuthal Electric Field (calculated for vacuum)



The radial current is causing distortion of the symmetry.

#### **Electric Field Induced by a Flat Coil Antenna**



Figure 4. Oscillatory velocity amplitude (a) and relative phase (b) obtained from emission spectroscopy (RF-MOS) at a pressure of p = 0.5 Pa. D L Crintea, D Luggenhölscher, V A Kadetov, Ch Isenberg and

U Czarnetzki, J. Phys. D: Appl. Phys. 41 (2008) 082003

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#### **Wave Equation for the Induced Field**

$$\Delta \vec{E} = \frac{\partial^2 \vec{E}}{\partial z^2} + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \vec{E} \right) \right) = \frac{\omega^2}{c^2} \vec{E} - i\omega \mu_0 \vec{j}.$$

- Two dimensional problem.
- Displacement current can be neglected since the plasma frequency is much higher than the RF frequency.
- Analytical solution only for a homogeneous plasma.

#### **Antenna Current Density**

Antenna current (N windings):

$$j = I \sum_{i=1}^{N} \delta(\vec{r} - \vec{r_i}) = \sum_{i=1}^{N} I \frac{\delta(r - r_i)}{r} \delta(z - z_0),$$

Approximating a large number of individual windings by a conducting disc leads to a series of Bessel functions:

$$j = I\delta(z) \frac{N}{R} \int_{0}^{R} \rho \frac{\delta(r-\rho)}{r} d\rho.$$
$$= \sum_{n=1}^{\infty} \tilde{j} J_1\left(\lambda_n \frac{r}{L}\right),$$

$$\widetilde{j} = -I\frac{N}{L}\frac{\pi}{\lambda_n}\frac{\left[J_0\left(\lambda_n\frac{R}{L}\right)H_1\left(\lambda_n\frac{R}{L}\right) - J_1\left(\lambda_n\frac{R}{L}\right)H_0\left(\lambda_n\frac{R}{L}\right)\right]}{J_1^{\prime 2}\left(\lambda_n\right)}.$$

#### **Plasma Current Density**

Local isotropic conductivity:

$$\vec{j} = \sigma \vec{E}$$
$$\sigma = \frac{e^2 n}{m \left(\nu + i\omega\right)}.$$

- For mean free paths of the order of the skin depth or larger the conductivity is no longer local.
- Then the product of conductivity and field becomes a convolution.

#### **Geometry and Boundary Condition**

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Conditions are set at the walls and the interfaces:

$$\frac{z=0}{(1)}$$

$$E_{v}|_{z=0} = E_{q}|_{z=0},$$

$$(2) \quad \frac{\partial E_{q}}{\partial z}\Big|_{z=0} - \frac{\partial E_{v}}{\partial z}\Big|_{z=0} = i\omega\mu_{0}j_{Antenne},$$

$$\frac{z=d}{(1)}$$

$$E_{q}|_{z=d} = E|_{z=d},$$

$$(2) \quad \frac{\partial E_{q}}{\partial z}\Big|_{z=d} = \frac{\partial E}{\partial z}\Big|_{z=d},$$
generally
$$(3) \quad E_{v}(z \to -\infty) = 0,$$

$$E(z = Z) = 0 \quad \text{und}$$
$$E(z = L) = 0.$$

(4)

(4)

Ζ

#### **Induced Field**

Radially Bessel functions, axially exponentials:

$$E(r,z) = \sum_{n=1}^{\infty} \tilde{A_n} \sinh\left(\sqrt{\left(\frac{\lambda_n}{L}\right)^2 + k_p^2} \left(Z - z\right)\right) J_1\left(\lambda_n \frac{r}{L}\right)$$

The coefficients are:

$$\tilde{A_n} = \frac{i\omega\mu_0 IN\frac{\pi}{\lambda_n} \frac{\left[J_0\left(\lambda_n \frac{R}{L}\right)H_1\left(\lambda_n \frac{R}{L}\right) - J_1\left(\lambda_n \frac{R}{L}\right)H_0\left(\lambda_n \frac{R}{L}\right)\right]}{J_1^{\prime 2}(\lambda_n)} e^{-\lambda_n \frac{d}{L}}}{\lambda_n \sinh\left(\sqrt{\left(\frac{\lambda_n}{L}\right)^2 + k_p^2} \left(Z - d\right)\right) + \sqrt{\lambda_n^2 + \left(k_p L\right)^2} \cosh\left(\sqrt{\left(\frac{\lambda_n}{L}\right)^2 + k_p^2} \left(Z - d\right)\right)}$$

- The major simplifying assumption in deriving this analytical solution is a homogeneous plasma density.
- With realistic spatial profiles solutions have to be obtained numerically.
### **Relevance of the Individual Terms**



Only very few terms contribute, actually only 2 or 3!

Dominik Winter, Master Thesis, RUB (2010)

### **Comparison with numerical integration**



### Self consistent numerical solution I



### Plasma density and ion flux

Dominik Winter, Master Thesis, RUB (2010)

## Self consistent numerical solution II



### Induced power and electron temperature

Dominik Winter, Master Thesis, RUB (2010)

# **Radial Profiles of T<sub>e</sub>**



Victor Kadetov, PhD Thesis, RUB (2004)

Collisional losses are strong in hydrogen and the energy is balanced locally. The electron temperature reproduces schematically the torus structure of the induced electric field (radial direction).

# Phase Resolved Velocity Distribution Function in an ICP





Argon ICP discharge, f = 13.56 MHz, P = 1 kW, p = 0.5 Pa. Accumulation over 90,000 shots (30 minutes) at each phase. Temporally independent EEPF, oscillating drift velocity.

D L Crintea, D Luggenhölscher, V A Kadetov, Ch Isenberg and U Czarnetzki, J. Phys. D: Appl. Phys. **41** (2008) 082003

### **Harmonically Oscillating Drift Velocity**



D L Crintea, D Luggenhölscher, V A Kadetov, Ch Isenberg and U Czarnetzki, J. Phys. D: Appl. Phys. **41** (2008) 082003

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### Self consistent numerical solution III



#### Electric field amplitude and phase

Dominik Winter, Master Thesis, RUB (2010)

### Measurement (B-dot probe)



### Very similar to simulation.



Dominik Winter, Master Thesis, RUB (2010)

### **Optically Measured Field Penetration**

Intensity and field are radially integrated and the axial dependence and time evolution are displayed as coloured contour plots.



Victor Kadetov, PhD Thesis, RUB (2004)

### **Measurement (optical modulation)**



### Very good agreement throughout.

Dominik Winter, Master Thesis, RUB (2010)

## **Transformer Model**



The plasma inductance and resistance are transformed to an effective antenna inductance and resistance.

$$R_{S} = R_{2} \cdot \frac{(\omega L_{12})^{2}}{R_{2}^{2} + (\omega L_{2})^{2}}$$
$$\omega L_{S} = \omega L_{1} - \omega L_{2} \cdot \frac{(\omega L_{12})^{2}}{R_{2}^{2} + (\omega L_{2})^{2}}$$

$$P_{diss} = \frac{1}{2} R_{S} \hat{I}_{RF}^{2}$$

### Low and High Density Operation

Transformed resistance:

Low density operation (skin depth >> plasma size):

High density (normal) operation (skin depth << plasma size):



$$R_P >> \omega L_P \implies R_S \propto \frac{1}{R_P} \propto n_e$$

$$R_P \ll \omega L_P \implies R_S \propto R_P \propto \frac{1}{\sigma_P \delta} \propto \frac{1}{\sqrt{n_e}}$$



# Dissipated Power and Operation Threshold

### low density:

$$P_{diss} \propto n_e \hat{I}_{RF}^2$$

high density:

$$P_{diss} \propto rac{1}{\sqrt{n_e}} \, \hat{I}_{RF}^2$$

power lost by the discharge (collisions, flux to the wall):

$$P_{loss} \propto n_e$$

 $P_{diss}$   $I_2 < I_1$  ICP operation  $I_3 < I_2$ 

P<sub>loss</sub>

electron density

- There is a minimum power (current) for ICP operation.
- Below this value, the discharge operates capacitively.

# **Capacitive Coupling by the Antenna**



- The quartz window and the sheath form a non-linear capacitive voltage divider between the antenna and the plasma.
- At high densities and/or low antenna voltages, the capacitive coupling is reduced strongly.

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# Effect of CCP-ICP Mode Transition on the Matching



 Divergence of the matching capacitance C<sub>1</sub> at the transition point to CCP operation.

Victor Kadetov, PhD Thesis, RUB (2004)

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# Effect on the Modulation of the Optical Emission

Discharge in hydrogen at p = 10 Pa: mode transition CCP to ICP at P = 150 W

Observation of the Balmer $-\alpha$  emission line (integrated spatially):

modulation 
$$= \frac{I(t)}{\overline{I}} \cdot 100\%$$



Victor Kadetov, PhD Thesis, RUB (2004)

# **Observation of the Mode Transition in the** Modulation of the Optical Emission



Pulsed mode: P = 300 W, f = 1 kHz, 1:1

- ICP discharges always ignite as CCPs.
- Then the density rises, the sheath shrinks, ۲ and the mode transition happens.

Victor Kadetov, PhD Thesis, RUB (2004)



- The mode transition between ICP and CCP has a strong effect on the ion energy distribution function.
- ICP: Quasi mono-energetic, low-energy ions from the floating potential sheath.
- CCP: High energy wide distributed ions from the RF sheath.

Victor Kadetov, PhD Thesis, RUB (2004)

# **Further Topics in ICPs**

- 1) Pulsed discharges and afterglow.
- 2) Electronegative discharges and instabilities.
- 3) Gas heating and electron pressure effects.
- 4) Atmospheric pressures.
- 5) Hybrid discharges ICP CCP.
- 6) Many applications.....

## ECR Discharges (and a little bit of Whistlers, Helicons)

# **Addition of a Weak Axial Magnetic Field**

Argon p = 0.1 Pa, B = 10 mT,  $\lambda_{me}$  = 1 m = 2 x diameter.





Helicon,

with

**B**-field

ICP, without B-field

# **RF Modulation Spectroscopy Upstream**



# **RF Modulation Spectroscopy Downstream**



- Modulation can still be observed downstream.
- A helicon wave propagates along the field lines.
- The diamagnetic drift  $(\text{grad}(T_e))$  displays the RF phase front.
- The phase front is bended due to the inhomogeneous dispersion by the density profile

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# Helicon Wave Magnetic Field (B-Dot Probe)



- The antenna field determines the  $B_z(r)$  wave field.
- Maxwell's equations then determine the Helicon B<sub>ω</sub>(r):

$$B_{\varphi}(r) \propto \frac{\partial}{\partial r} B_{z}(r)$$

Helicon (with static B-field)



# Electromagnetic Fields Propagating Along a Static Magnetic Field

- The wave vector k points along the field lines.
- All electric fields are perpendicular to the wave vector k.
- Two fundamental polarization states:
  - R-wave (rotating clockwise, with the electrons, "+")
  - L-wave (rotating counter clockwise, with the ions, "-")

$$\vec{\mathrm{E}} = \vec{\mathrm{E}}_0 e^{i(kz - \omega t)} \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

- *x* and *y*-components are 90° out of phase.
- The Lorentz force is important in this case:

$$m\,\dot{\vec{v}} = -\,\frac{e}{m} \left(\vec{E} + \vec{v} \times \vec{B}\right)$$

• Electrons rotate clockwise with the cyclotron frequency:

$$\omega_c = \frac{e B}{m_e}$$

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### **Determination of the Dispersion Relation**

Current density from the momentum equation:

$$-i\omega\,\vec{j} = \frac{\omega_{pe}^2}{c^2}\frac{\vec{\mathrm{E}}}{\mu_0} - \omega_c\,\vec{j}\times\vec{e}_z$$

Solution of the equation (determines also the conductivity):

$$-i\omega\mu_{0}\vec{j} = \frac{\omega_{pe}^{2}}{c^{2}}\frac{1}{1\mp\frac{\omega_{c}}{\omega}}\vec{E}$$

Insertion into the wave equation gives the dispersion relation:

$$-k^{2}\vec{E} + \frac{\omega^{2}}{c^{2}}\vec{E} = -i\omega\mu_{0}\vec{j} \implies -k^{2} + \frac{\omega^{2}}{c^{2}} = \frac{\omega_{pe}^{2}}{c^{2}}\frac{1}{1\mp\frac{\omega_{c}}{\omega}}$$

### **Structure of the Dispersion Relation**

Solving for  $k^2$  yields:

$$(ck)^{2} = \omega^{2} - \frac{\omega_{pe}^{2}}{1 \mp \frac{\omega_{c}}{\omega}} \quad "-":R - wave, "+":L - wave$$

Normalization by the electron plasma frequency helps identifying the characteristics:

$$k^{2} = \omega^{2} - \frac{1}{1 \mp \frac{\omega_{c}}{\omega}}$$
  
mit c k /  $\omega_{pe} \rightarrow k$ ,  $\omega / \omega_{pe} \rightarrow \omega$ ,  $\omega_{c} / \omega_{pe} \rightarrow \omega_{c}$ 

### **Cut-Offs and Resonances**

Cut-off frequencies (k = 0) are easily identified:

$$0 = \omega^{2} - \frac{1}{1 \mp \frac{\omega_{c}}{\omega}} \implies \omega = \omega_{R/L} = \frac{1}{2} \left( \pm \omega_{c} + \sqrt{\omega_{c}^{2} + 4} \right), 0$$
$$\implies \Delta \omega = \omega_{R} - \omega_{L} = \omega_{c}$$

- Three intersections with the  $\omega$ -axis.
- R-wave at  $\omega = 0$  and  $\omega = \omega_R$ .
- L-wave at  $\omega_L$ , in between the two cut-offs of the R-wave.
- Only the R-wave has a resonance  $(k \rightarrow \infty)$ .
- This resonance is at the cyclotron frequency  $\omega = \omega_c$ .
- Natural result with view on the electron gyration.

# **R- and L-Wave Dispersion**

- The dispersion relation is shown for  $\omega_c = 2 \omega_{pe}$ .
- Then the L-wave cut-off is below the R-wave resonance  $(\omega_{\rm c})$ .
- In case  $\omega_c < \omega_{pe}$ , the L-wave cut-off is between  $\omega_R$  and  $\omega_c$ .
- The R-wave has a bandgap between  $\omega_c$  and  $\omega_R$ .
- The L-wave has band-gap between 0 und ω<sub>L</sub>.
- For large k both waves behave like waves in vacuum with a phase velocity c.



# R-Wave Dispersion at Low $\omega$ and *k*: Whistler Waves

- At low frequencies only R-waves exist (cut-off for L-waves).
- The dispersion relation can then be expanded to first order  $(\omega << \omega_c)$  which provides a simple expression for the phase velocity:

$$\mathbf{v}_{ph} = \frac{\omega}{k} \approx c \frac{\sqrt{\omega \omega_c}}{\omega_{pe}}$$

- The phase velocity increases monotonically.
- Higher frequencies propagate faster and arrive earlier.
- If all frequency start at the same time a *x* = 0 and detection is after a distance *s*, each frequency arrives at its own time:

$$t = \int_{0}^{s} \frac{dx}{v_{ph}} \propto \frac{1}{\sqrt{\omega}} \Rightarrow \omega(t) \propto \frac{1}{t^{2}}$$

# **Natural Whistlers**

- Lightning flashes at the southern hemisphere excite whistler waves within a broad spectral range (kHz).
- Waves propagate along the earth magnetic field lines.
- Due to dispersion higher frequencies arrive earlier.



Stanford vfl group

http://www-pw.physics.uiowa.edu/mcgreevy/

- A falling glissando can be heard (coining the term "whistler").
- The time dependent frequency spectra scale like  $1/t^2$ .
- A fit "by eye" shows very good agreement (red lines).

# **Helicon and Whistler Dispersion**

- Helicon waves are whistler waves which are radially bounded.
- This requires a finite  $k_r$  and the dispersion relation changes.



- The difference is particularly pronounced at  $R < \lambda_z$ .
- For large radii the Helicon wave eventually merges into the normal whistler wave.

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# **Faraday Rotation**

- At high frequencies R- and L-waves both travel almost with the speed of light in vacuum.
- However, still the R-wave is a little faser.
- Therefore, with distance an increase phase difference develops. In second order in  $1/\omega$  this difference is:

$$\Delta \varphi = \varphi_L - \varphi_R = L \Delta k = \frac{L \omega_c}{c} \left(\frac{\omega_{pe}}{\omega}\right)^2 \propto B_0 n$$

Transformation of a linearly polarized wave into a left and right polarized wave:  $(\vec{e}_x + i\vec{e}_y) + (\vec{e}_x - i\vec{e}_y) = 2\vec{e}_x$ 

- The phase difference between the R- and L-components rotates the direction of polarization.
- This effect is called Faraday rotation.

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## **Application of Faraday Rotation**

The R- and L- parts are recombined after the wave has experienced a certain phase shift:

$$\vec{\mathbf{E}}_{R} + \vec{\mathbf{E}}_{L} = \frac{\mathbf{E}_{0}}{2} e^{i\varphi_{R}} \left[ \begin{pmatrix} 1 \\ i \end{pmatrix} + e^{i\Delta\varphi} \begin{pmatrix} 1 \\ -i \end{pmatrix} \right] = \mathbf{E}_{0} e^{i\varphi_{R} + \Delta\varphi/2} \begin{pmatrix} \cos(\Delta\varphi/2) \\ -\sin(\Delta\varphi/2) \end{pmatrix}$$

The initially only x-direction polarized wave has been rotated by an angle  $-\Delta \phi/2$  in direction of the y-axis.

**Applications:** 

- Measuring magnetic fields in astro physics.
- Determination of magnetic fields in fusion experiments.
- Building optical diodes for laser applications.

## **Electron Cyclotron Resonance (ECR)**

- ECR discharges take advantage of the resonance of the R-wave at the electron cyclotron frequency.
- For commercial applications they are generally operated at a microwave frequency of f = 2.45 GHz.
- This requires a magnetic field strength of B = 87.5 mT.
- The magnetic field is generally non-uniform.
- Waves are launched from the high field side ( $\omega_c > \omega$ ).



Kouichi Ono, Mutumi Tuda, Hiroki Ootera, and Tatsuo Oomori, *Pure & Appl. Chem.*, Vol. 66, No. 6, pp. 1327-1334, 1994.

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#### **Examples of ECR Sources**

- Ion source for highly charged ions, e.g. at accelerators.
- High density plasma source in the semiconductor industry.
- Satellite propulsion.





#### THE SACLAY (FRANCE) HIGH-CURRENT PROTON AND DEUTERON ECR SOURCE

Roth & Rau: commercial ECR source z.B. for semiconductor treatment, At P = 1 kW, n = a few  $10^{17}$  m<sup>-3</sup>. Pressure: p = 0,1 – 10 Pa.

# Control of the Resonance Location by the Magnetic Field



Two pictures show plasma density relates to the axial position of resonant magnetic field 875 G.

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### ECR Heating (ECRH) at the New STELLARATOR Wendelstein 7-X (W7-X)

Pulsed operation: 1 MW for 180 s, at f = 140 GHz

Figure shows a prototype of a so called "gyratron", a high power microwave source, during the development phase.



Fig. 2 Prototype Gyrotron ("Maquette") during test (by courtesy of FZK)

### Ion Cyclotron Resonance Heating (ICRH)

ICRH (Ion Cyclotron Resonant Heating) at the TOKAMAK Joint European Torus (JET): 23 – 57 MHz, up to 32 MW in 16 channels of 2 MW.





### **Collisionless Heating in ECR Discharges**

The equation of motion for a single particle is solved:  $\vec{\tau} = (\vec{\tau} + \vec{\tau} + \vec{p}) = \vec{\tau} (\vec{\tau} + \vec{r} + \vec{p}) = \vec{\tau} (\vec{r} + \vec{r} + \vec{r}) = \vec{\tau} \vec{r}$ 

 $m\dot{\vec{v}} = -e(\vec{E}_0 + \vec{v} \times \vec{B}), \ \vec{v}(z = z_0, t = 0) = \vec{v}_0$ 

- The time varying electric field *E* is assumed as homogeneous, i.e.it is only a function of time (corrections -> later).
- The static magnetic field *B* points only in the *z*-direction and is only a function of *z* (magnetic mirror effects are ignored).
- The aim is the calculation of the change in energy  $\Delta \varepsilon$  between the initial moment and a later moment.

### **Free Gyrating Particle**

• The initial condition can always be met by adding the solution for a free gyrating electron.

$$m\dot{\vec{v}}_{f} = -e\vec{v}_{h} \times \vec{B}, \ \vec{v}_{f}(z=z_{0},t=0) = \vec{v}_{0}$$

- This solution cancels out later when calculating the energy change and averaging over all possible initial phases.
- Solution for a homogeneous *B* field:

$$\vec{v}_f = v_0 \begin{pmatrix} \sin(\vartheta_0)\cos(\omega_c t + \varphi_0) \\ -\sin(\vartheta_0)\sin(\omega_c t + \varphi_0) \\ \cos(\vartheta_0) \end{pmatrix}, \quad \omega_c = \frac{eB}{m}$$

- If *B* varies with *z* only the time dependence changes.
- <u>Note:</u> lons gyrate counter clockwise and have a much higher mass (lower frequency).



#### Homogeneous **B**-Field

• The fields and initial conditions are now:

$$\frac{eB}{m} = \omega \vec{e}_z, \quad \vec{E} = E_0 \left( \vec{e}_x + i \vec{e}_y \right) e^{-i\omega t}, \quad \vec{v} \left( t = 0 \right) = 0$$

• The solution is:

$$\vec{v} = \hat{v} \, \varphi \begin{pmatrix} \cos(\varphi) \\ -\sin(\varphi) \\ 0 \end{pmatrix}, \quad \varphi = \omega t, \quad \hat{v} = \frac{e E_0}{m \, \omega}$$

- The velocity amplitude increases linearly with time!
- The energy increase quadratically:

$$\left<\Delta\varepsilon\right> = \frac{m\,\hat{v}^2}{2}\,\varphi^2$$

• Therefore, very high energies can be reached.

#### Inhomogeneous **B** Field

- An inhomogeneous *B* field is now approximated by a linear variation (first order expansion around the resonance point):  $\frac{e\vec{B}}{m} = \omega \left(1 \frac{z}{L}\right)\vec{e}_z$
- The solution is slightly more tricky but in the end effectively the linear term  $\varphi$  is replaced by an integral and some phase shift appears:  $\left(\cos(\varphi - \pi/4 + \theta(\varphi))\right)$

$$\vec{v} = \hat{v} \Psi(\varphi) \begin{bmatrix} cos(\varphi + \pi/4 + \theta(\varphi)) \\ 0 \end{bmatrix}, \quad \varphi = \omega t, \quad \hat{v} = \frac{eE_0}{m\omega}$$

• The integral reads:

$$\Psi(\varphi) = \left| \int_{0}^{\varphi} e^{i\Omega(\varphi')} d\varphi' \right| \approx \sqrt{2\pi \frac{L\omega}{v_{0z}}}, \quad \Omega(\varphi) = \frac{v_{0z}}{2L\omega} \varphi^{2} + \frac{z_{0}}{L} \varphi$$

• For typical parameters, the integral converges to a constant:  $L\omega >> v_{0z} \Rightarrow 10^{-2} m \, 10^{10} \, s^{-1} >> 10^6 \, m/s$ 

#### Energy Gain Along the z-Axis

Since the velocity along z is constant, phase  $\varphi = \frac{z}{L} = \frac{v_{0z}}{L\omega} \varphi - \frac{z_0}{L}$ and z-position can be directly converted:



Naturally, the width of the resonance zone is about L.

### Energy Gain in an Inhomogeneous B-Field

• The result for the energy gain is then:

$$\langle \Delta \varepsilon \rangle = \frac{m \hat{v}^2}{2} \left( 2\pi \frac{L\omega}{v_{0z}} \right) \text{ with } \frac{L\omega}{v_{0z}} >> 1$$

- The gain scales inversely with the initial velocity in z-direction, i.e. it is proportional to the transit time through the resonance zone.
- The power per unit area is obtained by multiplying with the flux density  $n v_{0z}$ :

$$S = \frac{\pi \left(e E_0\right)^2 Ln}{m \,\omega}$$

- Remarkably, this result is independent of the initial velocity.
- Since the frequency is very high collisions do not play a significant role up to a quite high pressure of about 100 Pa.

#### **Wave Equation Model**

- A correct description of the heating process must take the propagation and depletion of the wave into account.
- The collisonless wave equation is:

$$\frac{\partial^2 E_r}{\partial z^2} + k_0^2 \left( 1 - \frac{\omega_{pe}^2(z)}{\omega(\omega - \omega_{pe}(z))} \right) E_r = 0, \ k_0 = \frac{\omega}{c}$$

 Using again the linear B-field and constant plasma density approximation and some normalization, the equation reads:

$$\frac{\partial^2 E_r}{\partial \xi^2} + \left(1 - \frac{\eta}{\xi}\right) E_r = 0, \ \xi = zk_0, \ \eta = \frac{\omega_{pe}^2 L}{\omega c}$$

• A solution to this equation was found by K.G. Budden in1966 but it is rather complex.

K.G. Budden, Radio Waves in the Ionosphere, Cambrige University Press, UK 1966

#### **Reflection and Transmission**

 One of the most interesting aspect about the solution is that it allows also calculation of the reflection and transmission coefficients. These results are quite simple:

$$\frac{S_{abs}}{S_{inc}} = 1 - e^{-\pi\eta}, \ \frac{S_{trans}}{S_{inc}} = e^{-\pi\eta}, \ \frac{S_{ref}}{S_{inc}} = 0, \ \eta = \frac{\omega_{pe}^2 L}{\omega c} = \frac{\omega_{pe}^2}{\omega^2} k_0 L$$

- Quite unexpected, there is no reflection.
- Typical values are k<sub>0</sub> = 50 m<sup>-1</sup>, L = 0.1 m which leads to a moderate density condition:

$$\frac{\omega_{pe}^2}{\omega^2} \ge 0.2 \implies n \ge 1.5 \cdot 10^{10} \, cm^{-3}$$

### **Tunneling Through the Band Gap**

- The result of Budden implies that the wave is tunneling through the band gap which exists between  $\omega_c$  and  $\omega_R$ .
- Since  $\omega_c$  and z are interchangeable variables in the model, it is illustrative to re-plot the dispersion relation in an alternative representation.
- For low plasma densities the band gap is small and the wave can tunnel.
- Within the band gap, the wave is evanescent.



#### **Operation Parameters**

- A rough argument for the performance can be made from the global model.
- At low pressures:  $n \propto P_{abs}$ At high pressures:  $n \propto \sqrt{p} P_{abs}$
- This can be combined with the Budden result for the ratio of absorbed to incident power.
- The minimum can be found from the power-density relation and an expansion of the above result:

$$n/\varepsilon_T \propto P_{abs}$$

$$P_{abs} \approx \pi \eta P_{inc} \propto n P_{inc}$$

$$\Rightarrow \varepsilon_T \Rightarrow p_{\min}$$

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#### **Further Aspects**

• Doppler shift of the resonance:

$$\omega + k(z)v_z = \omega_c(z)$$

- This shifts the resonant magnetic field by typically a few percent.
- Heating at higher harmonics of the cyclotron frequency.
- Parametric instabilities and non-linear power absorption.
- Confinement of electrons in a mirror field.
- Effect of the chamber geometry on resonances (cavity effect at longer wavelengths or smaller chambers).
- Effects related to the transversal magnetic field.

#### **Microwave Discharges**

#### **Some Advantages of Microwave Sources**

- Simplicity of plasma generation at high (>100 W/cm<sup>3</sup>) and low (< 1 W/cm<sup>3</sup>) powers.
- Wide range of operating pressures (from 10 Pa up to atmospheric pressure).
- Control of the density profile by the antenna structure.
- Size of the discharge chamber can be variable.
- No electrodes.
- Large areas / volumes can be treated.
- High power sources at relatively low price available.



### **Sources Types I**

• Cavity and waveguide sources: Typically for flow-tube reactors with a quartz or ceramic tube.





Figure 2. Cavity microwave plasma generators



Figure 3. Waveguide microwave plasma generators

Y.A. Lebedev, Journal of Physics: Conference Series 257, 012016 (2010)

### **Sources Types II**

• Surface wave discharges: Flow tube as well as large area application.



**Figure 4.** Different designs of surface wave devices on the base of cylindrical (a, d) rectangular (b, c) and coaxial structures (e), duo-plasmaline ([10] Pr7-99) (f), large area surface wave plasma source (g) ([11] p. 163)

Y.A. Lebedev, Journal of Physics: Conference Series 257, 012016 (2010)

### **Sources Types III**

• Large area and large volume source.



Figure 5. (a) Waveguide holey-plate plasma generator ([11] p.175 )) and (b) SLAN-system (slotted antennas) ([10] Pr7-1)

Y.A. Lebedev, Journal of Physics: Conference Series 257, 012016 (2010)

#### **The SLAN Source**



#### **Ignition Characteristics**



Dirk Luggenhölscher, Dissertation RUB, 2004

### **Electric Fields During Ignition**



- Main ignition (strong density increase) occurs very fast.
- Prior to ignition the density changes slowly and is below the critical density so the field fills the discharge volume.

Dirk Luggenhölscher, Dissertation RUB, 2004

### **Density Decay in the Afterglow**

Argon dominated (90 % Ar, 10 %  $H_2$ ): p = 30 Pa , f = 200 Hz, 20 % on



- Source terms are only at the wall due to small skin depth.
- Plasma diffusion to the center plus recombination leads to hollow profiles.
- In the afterglow, relaxation to the basic diffusion mode.

Dirk Luggenhölscher, Dissertation RUB, 2004

#### **Density Profile and Pressure**

f = 200 Hz, 20 % on, <P> = 160 W



#### Simple model to describe the profiles

- stationary:
- radial diffusion
- recombination
- ionization << recombination (except edge)
- approximate solution:

$$\mathbf{n}(\mathbf{r}) \cong \mathbf{n}_0 \exp\left(\frac{1}{4}\left(\frac{\mathbf{r}}{\mathbf{r}_0}\right)^2 + \frac{1}{1152}\left(\frac{\mathbf{r}}{\mathbf{r}_0}\right)^4\right)$$

## At high pressures slow diffusion and strong recombination leads to edge profiles.

Dirk Luggenhölscher, Dissertation RUB, 2004

#### **Some Conclusions**

- At high frequencies the penetration of the electromagnetic wave into the plasma is a major issue.
- ECR discharges allow the wave to travel long distances before the energy is deposited within a relatively short range.
- In ICP and microwave discharges the skin depth is short and power is deposited in the vicinity of the antenna.
- Particularly microwave discharge have a skin depth of typically only one cm. However, power can be distributed by operation of a large number of antenna slots. These are often connected to one common resonator.
- Microwaves can propagate in the sheath between surfaces and the plasma via surface waves.
- All discharges can produce dense plasmas in the range of typically n = 10<sup>10</sup> cm<sup>-3</sup> to 10<sup>13</sup> cm<sup>-3</sup> and electron temperatures of a couple of eV and much higher in ECRs.

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